

Diagonal Forms of a Dual Scale Cosmology

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Abstract

A hybrid metric with off-diagonal temporal-radial behavior that was constructed to conveniently parameterized the early and late time behaviors of the universe is shown to have diagonal forms consistent with Robertson-Walker and deSitter geometries. The dynamics of the energy content of the cosmology as parameterized by the classical thermal fraction is briefly discussed as motivation for the comparison of the observables predicted by various microscopic models of the early evolution of the universe.

1 Introduction

There has been a considerable amount of interest in developing convenient frameworks for exploring the dynamics of the very early universe. In previous work[1, 2] a metric framework was developed that explicitly demonstrated the evolution of dual scales that could be associated with the microscopic and macroscopic behaviors of the cosmology. The temporal dynamics of these otherwise independent scales were shown to be connected only through the Einstein equation (in the absence of a cosmological constant). The metric, given by

$$ds^2 = -c^2 dt^2 + R^2(ct) \left(dr - \frac{r}{R_v(ct)} c dt \right)^2 + R^2(ct) (r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (1.1)$$

constrains the dynamics of an ideal fluid as expressed by

$$\frac{d}{dt} \rho = -\sqrt{\frac{24\pi G_N \rho}{c^4}} (P + \rho) \quad (1.2)$$

$$\rho = \frac{3c^4}{8\pi G_N} \left(\frac{\dot{R}}{R} + \frac{1}{R_v} \right)^2.$$

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The energy content is assumed to be of the form $\rho = \rho_v + \rho_{thermal}$ in terms of gravitational vacuum/condensate energy and thermal/classical energy, and the equations of state for the various components take the usual form $P_s = w_s \rho_s$.

If one defines the thermal (classical) fraction by $f(ct) \equiv \frac{\rho_{thermal}}{\rho}$ and the scale R_v is given by $\sqrt{\frac{8\pi G_N \rho_v}{3c^4}} = \frac{1}{R_v}$, the evolution equation satisfies

$$\begin{aligned} \frac{d}{dct} \rho &= -[(1 + w_v) + (w - w_v)f(ct)] \rho \sqrt{\frac{24\pi G_N}{c^4}} \rho, \\ \left(\frac{\rho_*}{\rho}\right)^{1/2} &= 1 + \frac{3}{2} \sqrt{\frac{8\pi G_N \rho_*}{3c^4}} \int_{ct_*}^{ct} [(1 + w_v) + (w - w_v)f(ct')] c dt'. \end{aligned} \quad (1.3)$$

The energy density and condensate scale are expected to have extreme values given by $\rho_I \Leftarrow \rho \Rightarrow \rho_\Lambda$ and $R_I \Leftarrow R_v \Rightarrow R_\Lambda$ for $0 \leftarrow ct \rightarrow \infty$. The initial and final states are taken to have a vanishing thermal fraction. The solution Eq. 1.3 can then be used to determine the temporal behavior of the thermal scale[2] R using Eq. 1.2.

2 Metric Diagonal Forms

For many, coordinates for which the metric form is diagonal provide the most intuitive construct of a given geometry. Therefore, diagonal forms will be constructed for the metric form in Eq. 1.1 using coordinate transformations on the coordinates (ct, r) . Two coordinate forms will be explored; one directly relating the dual scales to that of the Robertson-Walker (RW) geometry[3], and the other analogous to a deSitter geometry[4].

2.1 Radial coordinate transformation

A transformation on the radial coordinate will be sought to diagonalize the form in Eq. 1.1. Using Eq. 1.2, one is motivated to define the reduced scale factor \mathcal{R} , and require angular isotropy of the metric expressed in either coordinate system:

$$\frac{\dot{\mathcal{R}}}{\mathcal{R}} \equiv \frac{\dot{R}}{R} + \frac{1}{R_v} \quad , \quad R r = \mathcal{R} r_{RW} \quad . \quad (2.1)$$

A brief and straightforward calculation yields

$$R dr = \mathcal{R} \left[\frac{r_{RW}}{R_v} dct + dr_{RW} \right]. \quad (2.2)$$

Substitution into Eq. 1.1 gives

$$ds^2 = -c^2 dt^2 + \mathcal{R}^2(ct) \left[dr_{RW}^2 + r_{RW}^2 d\theta^2 + r_{RW}^2 \sin^2 \theta d\phi^2 \right]. \quad (2.3)$$

This demonstrates that the reduced scale parameter \mathcal{R} for coordinates (ct, r, θ, ϕ) corresponds to the standard Robertson-Walker scale parameter for coordinates $(ct, r_{RW}, \theta, \phi)$.

2.2 Temporal coordinate transformation

Next, a coordinate transformation on the temporal coordinate will be sought to diagonalize the metric form. For conciseness, the following forms will be defined:

$$\Delta(ct) \equiv \frac{R(ct)}{R_v(ct)} \quad , \quad d\tilde{t} \equiv \frac{c dt}{R(ct)} \quad . \quad (2.4)$$

Rewriting the metric in Eq. 1.1

$$ds^2 = R^2 \left[-d\tilde{t}^2 + (dr - r\Delta d\tilde{t})^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \quad (2.5)$$

all dimension is carried in the parameter $R(ct)$. The metric becomes diagonal under the temporal transformation

$$\begin{aligned} d\tilde{t} &= B(ct_{dS}, r) c dt_{dS} - \frac{r\Delta}{1-(r\Delta)^2} dr = \\ \frac{c dt}{R(ct)} &= B(ct_{dS}, r) c dt_{dS} + \frac{1}{2\Delta} \frac{\partial}{\partial r} \log(1 - (r\Delta)^2) dr, \end{aligned} \quad (2.6)$$

giving a metric of the form

$$ds^2 = R^2 \left(- \left[1 - (r\Delta)^2 \right] B^2 c^2 dt_{dS}^2 + \frac{dr^2}{[1 - (r\Delta)^2]} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \quad (2.7)$$

Thus, there is a space-time coordinate singularity at $r\Delta(ct) = 1$ corresponding to a coordinate horizon (if B is not singular on that surface).

The integrability condition for functions of the transformed coordinates constrains the form of the function B . A general form is given by

$$\begin{aligned} B(ct_{dS}, r) &= b(ct_{dS}) + \frac{\partial}{\partial ct_{dS}} \left[\frac{1}{2\Delta} \log(1 - (r\Delta)^2) \right] \\ &= b(ct_{dS}) + B(ct_{dS}, r) R \frac{\partial}{\partial ct} \left[\frac{1}{2\Delta} \log(1 - (r\Delta)^2) \right]. \end{aligned} \quad (2.8)$$

In the cosmology of interest, the scales satisfy dynamics in a manner that the form B will be non-singular. The metric in Eq. 2.7 corresponds to the usual deSitter coordinates $(ct_{dS}R, rR, \theta, \phi)$ with horizon scale R_v if the scale parameters R and R_v are constants and $b = 1$.

3 Conclusions

It has been demonstrated that the geometry described by the dual scale metric of interest is of the Robertson-Walker form without a cosmological constant, where the RW radial scale factor decomposes into components that are convenient for exploring the various extremum epochs of the cosmology. The early and late stages of the cosmology are conveniently described by macroscopic densities ρ_v and scales R_v using geometries similar to that of deSitter, while the intermediate behavior is essentially that of a classical Robertson-Walker geometry driven by thermal energy densities. Future work will utilize the transition behavior of this metric in manners similar to those in the references[5, 2] in an attempt to microscopically characterize the early form of the cosmology consistent with intermediate astrophysical observations.

References

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